MATHEMATICS

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AIM POINTS
MATHEMATICS
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XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XI (PQRS)

TRIGONOMETRIC FUNCTIONS

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. Following are some of the fundamental trigonometric identities :

(i) $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ or, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

(ii) $\cos\theta = \frac{1}{\sec\theta}$ or, $\sec\theta = \frac{1}{\cos\theta}$

(iii) $\cot\theta = \frac{1}{\tan\theta}$ or, $\tan\theta = \frac{1}{\cot\theta}$

(iv) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ or, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(v) $\sin^2\theta + \cos^2\theta = 1$

(vi) $1 + \tan^2\theta = \sec^2\theta$ or, $\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$

(vii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ or, $\operatorname{cosec}\theta - \cot\theta = \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

2. (i) $\sin(-\theta) = -\sin\theta$ or, $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

(ii) $\cos(-\theta) = \cos\theta$ or, $\sec(-\theta) = \sec\theta$

(iii) $\tan(-\theta) = -\tan\theta$ or, $\cot(-\theta) = -\cot\theta$

(iv) $\sin(90° - \theta) = \cos\theta$, $\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$, $\sec(90° - \theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(90° - \theta) = \sec\theta$, $\cot(90° - \theta) = \tan\theta$

(v) $\sin(90° + \theta) = \cos\theta$, $\cos(90° + \theta) = -\sin\theta$

$\tan(90° + \theta) = -\cot\theta$, $\cot(90° + \theta) = -\tan\theta$

$\sec(90° + \theta) = -\operatorname{cosec}\theta$, $\operatorname{cosec}(90° + \theta) = -\sec\theta$

(vi) $\sin(180° + \theta) = \sin\theta$, $\cos(180° + \theta) = -\cos\theta$

$\tan(180° + \theta) = -\tan\theta$, $\cot(180° + \theta) = -\cot\theta$

$\sec(180° + \theta) = -\sec\theta$, $\operatorname{cosec}(180° + \theta) = -\operatorname{cosec}\theta$

(vii) $\sin(270° - \theta) = -\cos\theta$, $\cos(270° - \theta) = -\sin\theta$

$\tan(270° - \theta) = \cot\theta$, $\cot(270° - \theta) = \tan\theta$

$\operatorname{cosec}(270° - \theta) = -\sec\theta$, $\sec(270° - \theta) = -\operatorname{cosec}\theta$

(viii) $\sin(270° + \theta) = -\cos\theta$, $\cos(270° + \theta) = \sin\theta$

$\tan(270° + \theta) = -\cot\theta$, $\cot(270° + \theta) = -\tan\theta$

$\operatorname{cosec}(270° + \theta) = -\sec\theta$, $\sec(270° + \theta) = \operatorname{cosec}\theta$

(ix) $\sin(360° - \theta) = -\sin\theta$, $\cos(360° - \theta) = \cos\theta$

$\tan(360° - \theta) = -\tan\theta$, $\operatorname{cosec}(360° - \theta) = -\operatorname{cosec}\theta$

$\sec(360° - \theta) = \sec\theta$, $\cot(360° - \theta) = -\cot\theta$

(x) Sine and Cosine functions and their reciprocal i.e. Cosecant and Secant functions are periodic functions with period 2π.

(xi)

Odd functions	**Even functions**
sine, tangent cotangent,	cosine, secant cosecant

EXERCISE-1

1. Theorem.
2. Trigonometric identities.
3. Prove the following identities :
   (i) $\sin^8\theta - \cos^8\theta = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$
   (ii) $\cot^4\theta + \cot^2\theta = \mathrm{cosec}^4\theta - \mathrm{cosec}^2\theta$
   (iii) $2\,\mathrm{sc}^2\theta - \sec^4\theta - 2\,\mathrm{cosec}^2\theta + \mathrm{cosec}^4\theta = \cot^4\theta - \tan^4\theta$
   (iv) $(\sin\theta + \mathrm{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = \tan^2\theta + \cot^2\theta + 7$
4. Prove the following identities :
   (i) $(1 + \cot\theta - \mathrm{cosec}\,\theta)(1 + \tan\theta + \sec\theta) = 2$
   (ii) $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1 + \sin\theta}{\cos\theta}$
5. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.
6. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, prove that $\cos\theta - \sin\theta\ \sqrt{2}\sin\theta$.
7. If $a\cos\theta + b\sin\theta = x$ and $a\sin\theta - b\cos\theta = y$, prove that $a^2 + b^2 = x^2 + y^2$.
8. If $\dfrac{ax}{\cos\theta} + \dfrac{by}{\sin\theta} = a^2 - b^2$, and $\dfrac{ax\sin\theta}{\cos^2\theta} - \dfrac{by\cos\theta}{\sin^2\theta} = 0$, prove that $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$
9. If $m^2 + m'^2 + 2mm'\cos\theta = 1$, $n^2 + n'^2 + 2nn'\cos\theta = 1$, and $mn + m'n' + (mn' + m'n)\cos\theta = 0$., prove that $m^2 + n^2 = \mathrm{cosec}^2\theta$.
10. If $a\cos\theta - b\sin\theta = c$, show that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$
11. Given that : $(1 + \cos\alpha)(1 + \cos\beta)(1 + \cos\gamma) = (1 - \cos\alpha)(1 - \cos\beta)(1 - \mathrm{cosy}\,\gamma)$. Show that one of the value of each member of this equality is $\sin\alpha\sin\beta\sin\gamma$.
12. If $\dfrac{\sin A}{\sin B} = p$ and $\dfrac{\cos A}{\cos B} = q$, find tan A and tan B.
13. If $\sec\theta + \tan\theta = p$, obtain the values of $\sec\theta$, $\tan\theta$ and $\sin\theta$ in terms of p.
14. If $\tan^2\theta = 1 - a^2$, prove that $\sec\theta + \tan^3\theta\,\mathrm{cosec}\,\theta = (2 - a^2)^{3/2}$. Also, find the value of a for which the above result holds true.
15. Prove that : $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4(\sin^6\theta + \cos^6\theta) - 13 = 0$
16. If $10\sin^4\alpha + 15\cos^4\alpha = 6$, find the value of $27\,\mathrm{cosec}^6\alpha + 8\sec^6\alpha$.

17. If x is any non–zero real number, show that cos θ and sin θ can never be equal to $x+\frac{1}{x}$.
18. Prove that :

$$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$$

19. Prove that :

$$\frac{\cos\theta}{1-\sin\theta} = \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$$

20. Prove that :

$$1-\frac{\sin^2\theta}{1+\cot\theta}-\frac{\cos^2\theta}{1+\tan\theta} = \sin\theta\cos\theta$$

21. If $\tan\theta = \frac{a}{b}$, show that $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2-b^2}{a^2+b^2}$
22. If $\tan^2\theta = 1 - e^2$, prove that $\sec\theta + \tan^3\theta\ \text{cosec}\,\theta = (2-e^2)^{3/2}$.
23. Find the values of cos θ and tan θ if $\sin\theta = -\frac{3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$.
24. If $\cos\theta = -\frac{1}{2}$ and $\pi < \theta < \frac{3\pi}{2}$, find the value of $4\tan^2\theta - 3\,\text{cosec}^2\theta$.
25. Prove that :

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \begin{cases} \sec\theta - \tan\theta & ,\text{if } -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\ -\sec\theta + \tan\theta & ,\text{if } \frac{\pi}{2} < \theta < \frac{3\pi}{2} \end{cases}$$

26. Prove that :

$$\sqrt{\frac{1+\cos\theta}{1\_\cos\theta}} = \begin{cases} \text{co}\sec\theta - \cot\theta & ,\text{if } 0 < \theta < \pi \\ -\text{co}\sec\theta + \cot\theta & ,\text{if } 0 < \theta < 2\pi \end{cases}$$

27. If $\sin\theta = \frac{12}{13}$ and θ lies in the second quadrant, find the value of sec θ + tan θ.
28. If $\sin\theta = \frac{3}{5}$ $\tan\phi = \frac{1}{2}$ and $\frac{\pi}{2} < \theta < \pi < \phi < \frac{3\pi}{2}$, find the value of $8\tan\theta - \sqrt{5}\sec\phi$.
29. If $\cos\theta = -\frac{3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$ find the values of other five trigonometric functions and hence evaluate $\frac{\cos\text{ec}\,\theta + \cot\theta}{\sec\theta - \tan\theta}$.
30. Prove that : cos 150° cos 330° + sin 390° cos 120° = –1.
31. If A, B, C, D are angles of a cyclic quadrilateral, proe that : cos A + cos B + cos C + cos D = 0.
32. In any quadrilateral ABCD, prove that :

    (i) sin(A + b) + sin (C + D) = 0 (ii) cos(A + B) = cos (C + D)

33. Prove that :

(i) $\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\dfrac{\pi}{2}+x\right)}=\cot^2 x$

(ii) $\cos\left(\dfrac{3\pi}{2}+x\right)\cos(2\pi+x)\left\{\cot\left(\dfrac{3\pi}{2}-x\right)+\cot(2\pi+x)\right\}=1$

34. Prove that : $\sin^2\sin^2\dfrac{\pi}{18}+\sin^2\dfrac{\pi}{9}+\sin^2\dfrac{7\pi}{18}+\sin^2\dfrac{4\pi}{9}=2$

35. If A, B, C, D be the angle of a cyclic quadrilateral, takenin order, prove that
$\cos(180° - A) + \cos(180° + C) - \sin(90° + D) = 0$.

36. Proe that :

(i) $\tan 720° - \cos 270° - \sin 150° \cos 120° = \dfrac{1}{4}$

(ii) $\sin 780° \sin 480° + \cos 120° \sin 150° = \dfrac{1}{2}$

(iii) $\sin 780° \sin 120° + \cos 240° \sin 390° = \dfrac{1}{2}$

(iv) $\sin 600° \cos 390° + \cos 480° \sin 150° = -1$

(v) $\tan 225° \cot 405° + \tan 765° \cot 675° = 0$

EXERCISE-2

1. If $\sec\theta = x + \dfrac{1}{4x}$, then $\sec\theta + \tan\theta =$

(a) $x, \dfrac{1}{x}$ (b) $-\dfrac{1}{2x}, 2x$ (c) $2x$ (d) $2x, \dfrac{1}{2x}$

2. If $\dfrac{\pi}{2}<\theta<\dfrac{3\pi}{2}$, then $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}$ is equal to

(a) $\sec\theta - \tan\theta$ (b) $\sec\theta + \tan\theta$ (c) $\tan\theta - \sec\theta$ (d) none of these

3. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, then $x^2 + y^2 + z^2$ is independent of

(a) θ, ϕ (b) r, θ (c) r, ϕ (d) r

4. If $\tan\theta = -\dfrac{1}{\sqrt{5}}$ and θ lies in the IV quadrant, then the value of $\cos\theta$ is

(a) $\dfrac{\sqrt{5}}{\sqrt{6}}$ (b) $\dfrac{2}{\sqrt{6}}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{\sqrt{6}}$

5. $\text{Sin}^6 A + \cos^6 A + 3 \sin^2 A \cos^2 A =$

(a) 0 (b) 1 (c) 2 (d) 3

6. If θ is an acute angle and $\tan\theta = \frac{1}{\sqrt{7}}$, then the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$ is

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{5}{4}$

7. $\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9} =$

(a) 1 (b) 4 (c) 2 (d) 0

8. If $x \sin 45° \cos^2 60° = \frac{\tan^2 60° \text{cosec} 30°}{\sec 45° \cot^{2°} 30°}$, then x =

9. If cosec A + cot A = $\frac{11}{2}$, then tan A =

(a) $\frac{21}{22}$ (b) $\frac{15}{16}$ (c) $\frac{44}{117}$ (d) $\frac{117}{43}$

10. If $\tan\theta + \sec\theta = e^x$, then $\cos\theta$ equals

(a) $\frac{e^x + e^{-x}}{2}$ (b) $\frac{2}{e^x + e^{-x}}$ (c) $\frac{e^x - e^{-x}}{2}$ (d) $\frac{e^x - e^{-x}}{e^x + e^{-x}}$